COMPARISON OF HOLT'S METHOD AND WINTER'S METHOD FOR FORECASTING QUALITY

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ABSTRACT

Forecasting involves making projection about future performance on the basis of historical and current data. Forecasting is widely used for planning decisions in various domains of management. Every business, government agency, non-profit organization needs timely forecasts. This paper studies forecasting methods. Among the forecasting methods, this paper emphasized the Holt (Exponential smoothing with trend) and Winter (Exponential smoothing with trend and seasonality). This system examines Holt and Winter parameter estimation techniques. The aim of this system is to compare their forecast quality via the criteria of forecasting using monthly time series data of child diseases and presents forecasts for future. Comparison statistics and statistics of errors for these methods are examined.

1. INTRODUCTION

Many of the forecasting techniques used currently were developed in the nineteenth century. By contrast, some techniques were developed and have received attention only recently. With the development of more sophisticated forecasting techniques along with the use of personal computers and associated software, forecasting has received more and more attention. Forecasters could only attempt to make the inevitable errors as small as possible. Despite inherent in accuracies, forecasting is necessary because all organizations must make decisions under uncertainty [1]. The need for quick, reliable, simple and short or medium term forecasts of various times series is often encountered in economic and business environment. Holt's method and Winter's method provide a comprehensive, simple, accurate and application solution to this problem. Their accuracy can be deduced from their forecast errors and other forecast criteria, referred as comparison statistics [2].

2. TIME SERIES ANALYSIS

Time Series (TS) is a sequence of observations ordered in time. Mostly the observations are collected at equally spaced, discrete time intervals. When there is only one variable upon which observations are made then we call a single time series or more specifically a univariate time series. A basic assumption in any time series analysis/ modeling is that some aspects of the past pattern will continue to remain in the future. If time series models are put to use, say, for instance, for forecasting purposes, then they are especially applicable in the 'short term'. Here it is tacitly assumed that information about the past is available in the form of numerical data. Ideally, at least 50 observations are necessary for performing TS analysis/modeling.

Time series models have advantages in certain situations. They can be used more easily for forecasting purposes because historical sequences of observations upon study variables are readily available from published secondary sources. These successive observations are statistically dependent and time series modeling is concerned with techniques for the analysis of such dependencies. Thus in time series modeling, the prediction of values for the future periods is based on the pattern of past values of the variable under study, but not generally on explanatory variables which may affect the system. There are two main reasons for resorting to such time series models. First, the system may not be understood, and even if it is understood it may be extremely difficult to measure the cause and effect relationship, second, the main concern may be only to predict what will happen and not to know why it happen. Many a time, collection of information on causal factor affecting the study variables may be cumbersome impossible and hence availability of long series data on explanatory variables is a problem. In such situations, the time series models are a boon for forecasters [3].

There are two main goals of time series analysis: (1) identifying the nature of the phenomenon represented by the sequence of observation, and (2) forecasting (predicting future values of the time series variable). Both of the goals require that the pattern of observed time series data is identified and formally described. Most time series pattern can be described in terms of three basic classes of components: trend, periodic components (e.g. seasonality), and noise [4].

3. FORECASTING METHODS

There are two main types of forecasting method: quantitative and qualitative. The quantitative forecasting methods include time series methods and causal methods. A time series method is appropriate when the historical data are restricted to past values of the variable being forecast.

The smoothing methods are appropriate for a stable time series, that is, one that exhibits no significant trend, cyclical, or seasonal effects. There are three smoothing methods (moving average, weighted moving average, and exponential smoothing method). When a time series consists of random fluctuation round a long-term trend line, a linear equation may be used to estimate the trend. When seasonal effects are present, seasonal indexes can be computed and used to deseasonalize the data and to develop forecasts. When seasonal and long-term trend effects are present, a trend line is fitted to the deseasonalized data; the seasonal indexes are used to adjust the trend projections.

Causal forecasting methods are based on the assumption that the variable we are trying to forecast exhibits a cause-effect relationship with one or more other variables. For instance, the sales volume for many products is influenced by advertising expenditures, so regression analysis may be used to develop an equation showing how these two variables are related. Then, once the advertising budget has been set for the next period, we could substitute this value into the equation to develop a prediction or forecast of the time sale volume for the next period.

Quantitative methods generally involve the use of expert judgment to develop forecasts. For instance, a panel of experts might develop a consensus forecast of the prime rate for a year from now. The advantage of quantitative procedures is that they can be applied when the information on the variable being forecast cannot be qualified and when historical data either are not applicable or variable [5].

4. EXPONENTIAL SMOOTHING

Exponential smoothing is a procedure for continually revising a forecast in the light of more recent experience. Exponential Smoothing assigns exponentially decreasing weights as the observation get older. In other words, recent observations are given relatively more weight in forecasting than the older observations [6].

4.1. Holt's Method: Exponential Smoothing with Trend

If a time series exhibits a linear trend (and no seasonality), Holt's method often yields good forecasts. At the end of the *t*th period, Holt's method yields an estimate of the best level (L_t) and the per-period trend (T_t) of the series. For example, suppose that L_{20}=20 and T_{20}=2. This means that after observation x_{20}, we believe that the base level of the series is 20 and that the base level is increasing by two units per period. Thus, five periods from now, we estimate that the base level of the series will equal 30.

After observation x_t, equations (1) and (2) are used to update the base and trend estimates α and β are smoothing constants, each between 0 and 1.

$$L_t = \alpha x_t + (1- \alpha) (L_{t-1} + T_{t-1}) \quad (1)$$

$$T_t = \beta (L_t - L_{t-1}) + (1- \beta) T_{t-1} \quad (2)$$

To compute L_t, we take a weighted average of the following two quantities:

1. x_t, which is an estimate of the period *t* base level from the current period
2. $L_{t-1} + T_{t-1}$, which is an estimate of the period *t* base level based on previous data

To compute T_t, we take a weight average of the following two equations:

1. An estimate of trend from the current period given by the increase in the smoothed base from period *t*-1 to period *t*
2. T_{t-1}, which is our previous estimate of the trend

As before, we define $f_{t,k}$ to be the forecast for x_{t+k} made at the end of period *t*. Then

$$F_{t,k} = L_t + kT_t \quad (3)$$

To initialize Holt's method, we need an initial estimate (call it L_0) of the base and an initial estimate (call it T_0) of the trend. We might set T_0 equal to the average monthly increase in the time series during the previous year, and we might see L_0 equal to last month's observation [7].

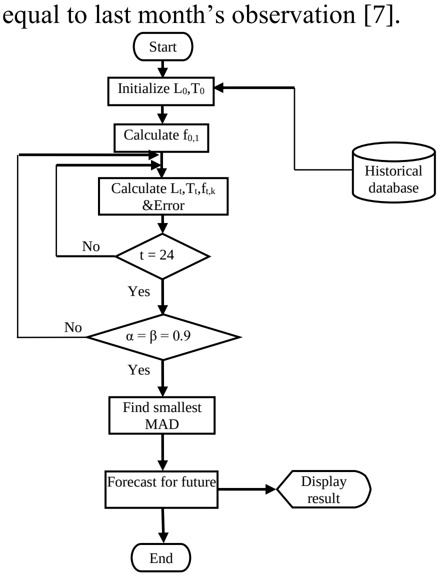

Figure 1. Flow chart of Holt's method

4.2. Winter's Method:Exponential Smoothing with Seasonality

The appropriately named Winter's method is used to forecast time series for which trend and seasonality are present [7]. Winter's method is capable of handling periods of different lengths. Every period length from yearly data to weekly data can be handled. The minimum input requirement is four years; thus, for monthly data, at least forty-eight values must be available [8].

To describe Winter's method, we require two definitions. Let c = the number of periods in the length of the seasonal pattern (c = 4 for quarterly data, and c = 12 for monthly data). Let s_t be an estimate of a seasonal multiplicative factor for month *t*, obtained after observing x_t. For instance, suppose month 7 is July and $s_7 = 2$. Then after observation month 7's air conditioner sales, we believe that July air conditioner sales will (all other things being equal) equal twice the sales expected during an average month. If month 24 is December, and $s_{24} = 0.4$, then after observing month 24 sales, we predict that December air conditioner sales will be 40% of the expected sales during an average month. In what follows, L_t and T_t have the same meaning as they did in Holt's method. Each period, L_t, T_t, and s_t are updated (in that order) by using Equation (4) – (6). Again, α, β and γ are smoothing constants, each of which is between 0 and 1.

$$L_t = \alpha\,(x_t/s_{t-c}) + (1-\alpha)\,(L_{t-1}+T_{t-1}) \quad (4)$$

$$T_t = \beta\,(L_t-L_{t-1}) + (1-\beta)\,T_{t-1} \quad (5)$$

$$s_t = \gamma\,(x_t/L_t) + (1-\gamma)s_{t-c} \quad (6)$$

Equation (4) updates the estimate of the series base by taking a weighted average of the following two equations:

1. $L_{t-1}+T_{t-1}$, which is our base level estimate before observation x_t
2. The deseasonalized observation x_t/s_{t-c}, which is an estimate of the basic obtained from the current period

Equation (6) updates the estimate of month *t*'s seasonality by taking a weighted average of the following two quantities:

1. Our most recent estimate of month *t*'s seasonality (s_{t-c})
2. x_t/T_t, which is an estimate of month *t*'s seasonality, obtained from the current month

At the end of period *t*, the forecast ($f_{t,k}$) for month t+k is given by

$$f_{t,k} = (L_t + kT_t)\,s_{t+k-c} \quad (7)$$

Thus, to forecast the value of the series during period t + k, we multiply our estimate of the period t + k base ($L_t + kT_t$) by our most recent estimate of month (t + K)'s seasonality factor (s_{t+k-c}).

To obtain good forecasts with Winter's method, we must obtain good initial estimates of base, trend, and all seasonal factors [7]. Let

L_0 = estimate of base at beginning of month 1
T_0 = estimate of trend at beginning of month 1
s_{-11} = estimate of January seasonal factor at beginning of month 1
s_{-10} = estimate of February seasonal factor at beginning of month 1
s_0 = estimate of December seasonal factor at beginning of month 1

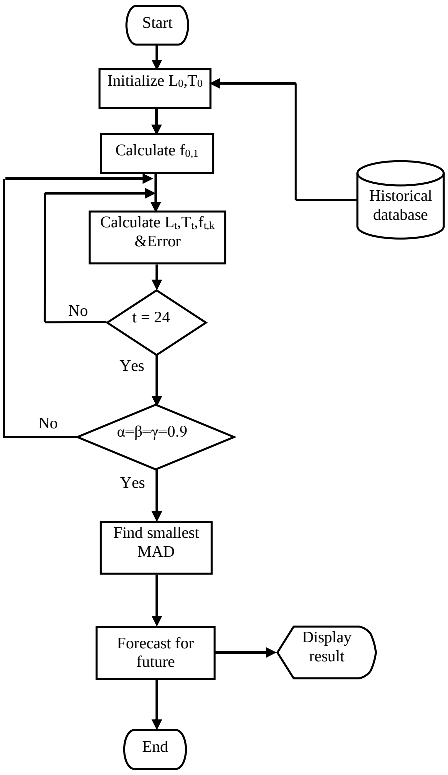

Figure 2. Flow Chart of Winter's Method

4.3. Error Measure

The accuracy of various forecasting methods typically requires comparisons across many time series. At the main aim in forecasting is to increase the accuracy of the forecasts, error measures are very important from a forecaster's point of view [9]. One common method to measure overall forecast error is the mean absolute deviation (MAD). This value is computed by taking the sum of the absolute values of the individual forecast errors and dividing by the sample size, the number of forecast periods. The formula is as follow:

$$\text{MAD}=\frac{1}{n}\;{}_{t=1}^{\;n} \qquad (8)$$

In this formula, Y_t is actual value in time period t, F_t is forecast value in time period t and n is number of periods [1].

5. SYSTEM DESIGN

There are three main parts in this system. "Forecast with Holt's Method", "Forecast with Winter's Method" and "Comparison of Holt's method Vs. Winter's method". If the user wants to forecast with Holt's method or Winter's method, the desired method can be selected.

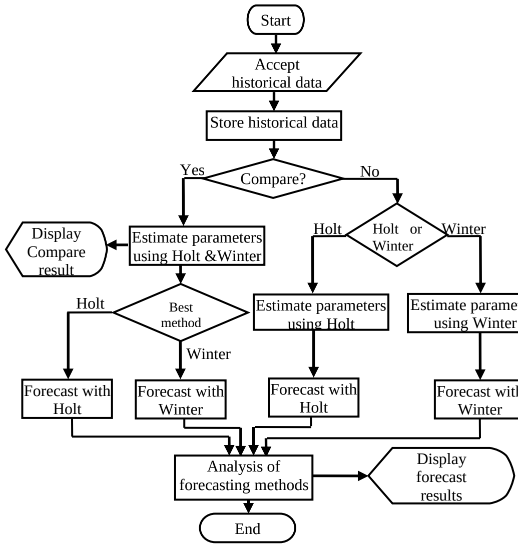

Figure 3. System Design

5.1. Comparison of Holt's method& Winter's method

In comparison of Holt's method Vs. Winter' method, if the user chooses disease and age group, the system gives the following comparison results in Table 1. Moreover, runtime comparison is done using with the same specification.

$\sum |Y_t - F_t|$ 2

Table 1. Comparison results of Holt's method and Winter's method for Asthma

Age Group		Holt Method	Winter Method
Under 5 year	α	0.9	0.6
	β	0.3	0.1
	γ	-	0.9
	MAD	18.8846	8.75553
	Calculation Time(sec)	0.004	0.051
Between 5 and 10 year	α	0.9	0.7
	β	0.3	0.1
	γ	-	0.9
	MAD	18.9109	10.7994
	Calculation Time(sec)	0.002	0.017
Under 14 year	α	0.9	0.8
	β	0.2	0.1
	γ	-	0.2
	MAD	18.4826	11.6314
	Calculation Time(sec)	0.001	0.017

In Table 1, the comparison results of Holt's method and Winter's method are shown. Comparison results are MAD, their parameters that give the smallest MAD and their calculation times.

At under 5 year, parameters (α = 0.6, β = 0.1, γ = 0.9) that give the smallest MAD of Winter's method are the most appropriate parameters for this method. MAD of Winter's method (8.75553) is smaller than MAD of Holt's method (18.8846). Thus, Winter's method is the best method for under 5 year of Asthma.

In figure 4, the graph of historical monthly data and forecast data is illustrated for Winter's method using α = 0.6, β = 0.1, γ = 0.9. Forecasting for future is made with these parameters. The result of forecasting for next 8^{th} month is 197.40.

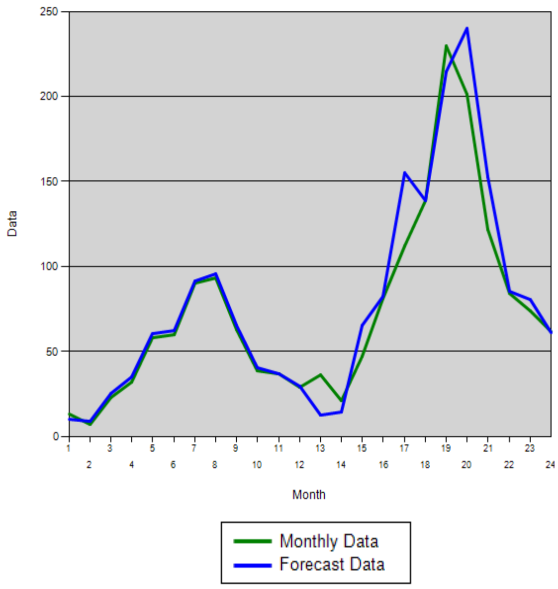

Figure 4. Winter's method forecasting (α = 0.6, β = 0.1, γ = 0.9)

6. CONCLUSION

Forecasting a time series is the process of projecting a time series into the future. There are two important types of forecasting methods: extrapolation methods and causal forecasting methods. Extrapolation methods are used to forecast future values of a time series from past values of a time series. In this method, past patterns and trends will continue in future months. Causal forecasting methods attempt to forecast future values of a variable by using past data to estimate the relationship between the dependent variable and one or more independent variables.

In this system, two extrapolation methods are used. Their forecast qualities are measured with Mean Absolute Deviation (MAD). When a time series has no seasonality and exhibits some form of trend, Holt's method gives a good forecast. If a time series has both trend and seasonal components, Winter's method gives better forecast than Holt's method. But Holt's method is more simplicity and less computation time than Winter's method for any time series data. This system has some limitations. In forecasting for future, Holt's method and Winter's method require historical time series data at least four years. Moreover, historical dataset must be time series dataset.

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